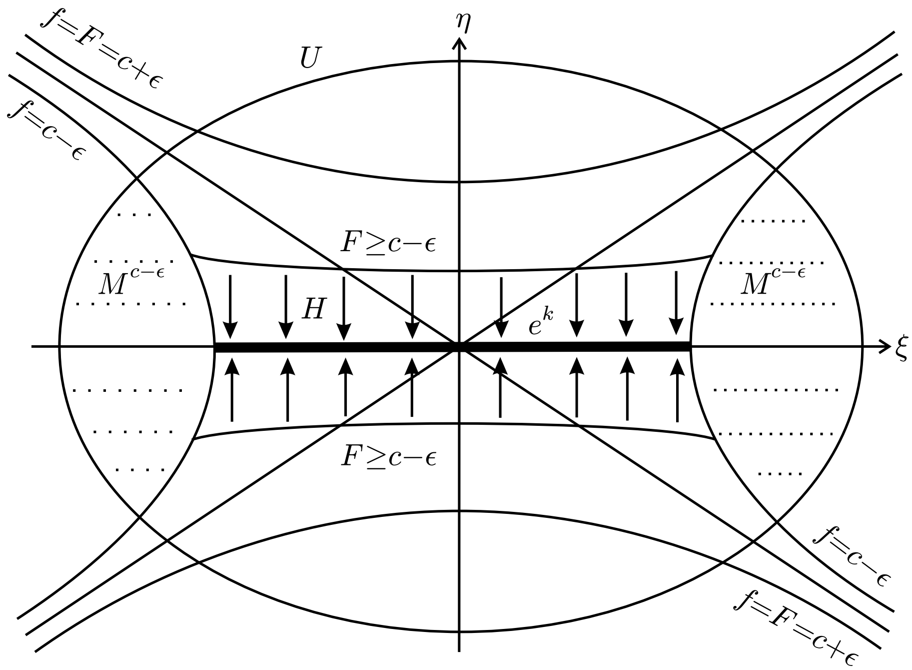

Fact 4: If H is the closure of $F^{-1}((-\infty, c-\varepsilon]) - M^{c-\varepsilon}$, then $M^{c-\varepsilon} \cup e^k$ is a deformation retract of $M^{c-\varepsilon} \cup H$. Since we have already shown that $F^{-1}((-\infty, c-\varepsilon]) = M^{c-\varepsilon} \cup H$ is a deformation retract of $M^{c+\varepsilon}$, this shows that $M^{c-\varepsilon} \cup e^k$ is a deformation retract of $M^{c+\varepsilon}$ (which is in turn a deformation retract of M^b by Theorem 3.20).

We construct the deformation retraction $r_t : M^{c-\varepsilon} \cup H \to M^{c-\varepsilon} \cup H$ as the identity outside of U. Inside U the retraction r_t is defined as follows. In region I, i.e. the set $\{x \in U | \, \xi^2 \leq \varepsilon\}$, r_t is defined by

$$r_t(x_1, \dots, x_m) = (x_1, \dots, x_k, tx_{k+1}, \dots, tx_k)$$

for $0 \leq t \leq 1$. Here r_1 is the identity and r_0 maps region I to e^k. Also, r_t maps $F^{-1}((-\infty, c-\varepsilon])$ into itself since $\frac{\partial F}{\partial s} > 0$.

In region II, i.e. $\{x \in U | \, \varepsilon \leq \xi^2 \leq \eta^2 + \varepsilon\}$, r_t is defined by

$$r_t(x_1, \dots, x_m) = (x_1, \dots, x_k, \lambda_t x_{k+1}, \dots, \lambda_t x_m)$$

where $\lambda_t \in [0, 1]$ is defined by

$$\lambda_t = t + (1-t)\sqrt{(\xi^2 - \varepsilon)/\eta^2}.$$

Thus r_1 is the identity, and r_0 maps region II into $M^{c-\varepsilon}$ since

$$\begin{aligned} f(r_0(x_1, \dots, x_m)) &= f(x_1, \dots, x_k, \sqrt{(\xi^2 - \varepsilon)/\eta^2} x_{k+1}, \dots, \sqrt{(\xi^2 - \varepsilon)/\eta^2} x_m) \\ &= c - \varepsilon. \end{aligned}$$