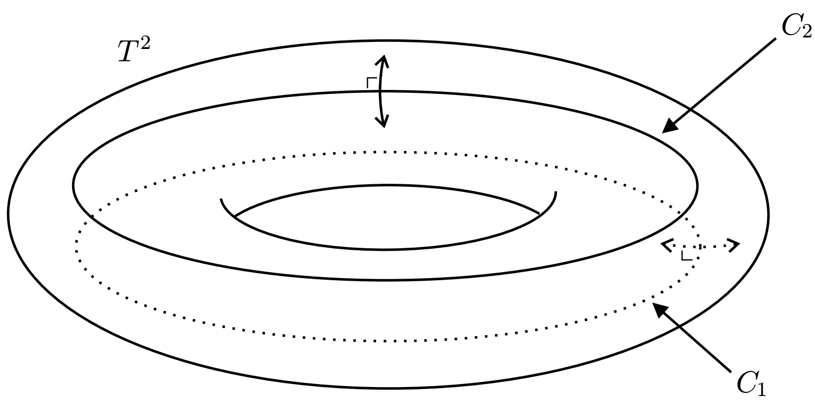
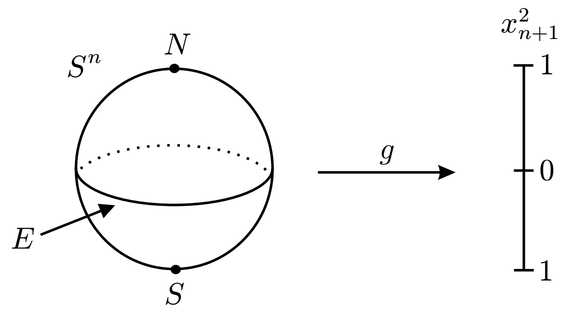
Example 3.47 The height function $f : T^2 \to \mathbb{R}$ of a doughnut lying on a dinner plate is a Morse-Bott function. There are two critical submanifolds, C_1 and C_2, each diffeomorphic to the circle S^1. It is easy to check that the second derivative of f in the direction perpendicular to the critical submanifolds is non-zero, and hence $H_p^\nu(f)$ is non-degenerate for all $p \in \text{Cr}(f)$.



Example 3.48 Any constant function $f \equiv c \in \mathbb{R}$ is a Morse-Bott function since $\text{Cr}(f) = M$ and $\nu_* C = \emptyset$. The trivial remark that $f \equiv 0$ is a Morse-Bott function turns out to be very useful!

Example 3.49 The square of the height function on S^n considered in Example 3.5 is a Morse-Bott function. Notice that even in this simple example the critical submanifolds are of different dimensions, two of the critical submanifolds (N and S) have dimension 0 and the other critical submanifold has dimension $n-1$. This is not uncommon. There is no reason to expect that the critical submanifolds of a Morse-Bott function will be of the same dimension.



Example 3.50 If $\pi : E \to M$ is a smooth fiber bundle over a smooth manifold M and $f : M \to \mathbb{R}$ is a Morse-Bott function, then the composite $f \circ \pi : E \to \mathbb{R}$ is a Morse-Bott function.