THREE APPROACHES TO MORSE-BOTT HOMOLOGY: ERRATUM CONCERNING THE ORIENTATION ASSUMPTIONS

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Corrections

The statement of Theorem 3.5 (Morse-Bott Inequalities) in the paper says,

"...assume that all the critical submanifolds of f are orientable."

This should be corrected to,

"...assume that all the negative normal bundles of the critical submanifolds of f are orientable."

The last paragraph of Section 3 should be replaced with the following:

Thus, Bott's version of the Morse-Bott inequalities reduces to the conclusion of Theorem 3.5 when the disk bundles given by the negative part of the gradient flow near the submanifolds are orientable. That is, the tangent space of M along a critical submanifold C has a decomposition

$$T_*M = T_*C \oplus \nu_*^- C \oplus \nu_*^+ C,$$

and it is the assumption that the bundle $\nu_*^- C$ is orientable that allows one to conclude that the Betti numbers with local coefficients in the orientation bundle used by Bott reduce to the Betti numbers considered in Theorem 3.5. Unfortunately, Banyaga and Hurtubise's paper incorrectly stated the assumption that T_*M and T_*C are orientable, instead of the assumption that $\nu_*^- C$ is orientable. These two conditions are distinct when $\nu_*^+ C$ is not orientable.

Acknowledgments

I would like to thank Thomas Rot for pointing out the mistake in the orientation assumptions and providing a nice counterexample of a Morse-Bott function on $\mathbb{RP}^5$ with two isolated critical points and a critical submanifold diffeomorphic to $\mathbb{RP}^3$.

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